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Local Dispersion Insert: The γ_T Knob for Accelerators

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Abstract

The Local Dispersion Insert is a novel three-quadrupole cell which produces local dispersion and beta waves while leaving all other beam parameters unchanged. The obtained dispersion bump can be effectively used to control the transition energy and the so-called γ_T jump of an accelerator. Since the induced beta wave is local and the tune shift is zero, the Local Dispersion Insert is an extremely efficient cell, typically requiring only a small number of magnets in order to achieve a γ_T jump. Since the desired γ_T jump can be obtained by locally *decreasing* the dispersion, the well known problems which accompany the conventional γ_T jump schemes, like large maximal dispersion, transverse emittance growth *etc.* are absent. The paper contains three applications of the Local Dispersion Insert: the γ_T jump in the Main Injector, the η adjustment in the Antiproton Debuncher (both at Fermi National Accelerator Laboratory), and the γ_T jump in the Relativistic Heavy Ion Collider at Brookhaven National Laboratory.

1 Introduction

It is often desirable to change the value of the frequency–momentum factor η of a synchrotron, which determines the relation between the momentum of a particle and its revolution frequency:

$$\eta = \frac{\Delta f/f}{\Delta p/p}.$$

η plays a crucial role in the longitudinal beam dynamics and in the stochastic cooling of the beam. In terms of the accelerator parameters and the beam energy, η can be expressed as

$$\eta = \frac{1}{C}\oint_C \frac{D_x(s)}{\rho(s)}ds - \frac{1}{\gamma^2}, \tag{1}$$

where C is the machine circumference, $D_x(s)$ the dispersion function of the accelerator, $\rho(s)$ the local bending radius, and γ the Lorentz parameter of the beam. Changing η of an accelerator thus translates into changing its dispersion function D_x. The integral term in Eq. (1) is usually denoted $\frac{1}{\gamma_T^2}$ where γ_T (the *transition* γ) has the meaning of the beam energy at which η changes sign.

The γ_T *jump* which secures speedy transition crossing is probably the most widely known example of this kind. The transition occurs at the energy $\gamma = \gamma_T$ and is characterized by a multitude of problems.[1] Rapid crossing of transition alleviates (or eliminates) these problems. In order to achieve the γ_T jump, the dispersion function must be temporarily changed such that $\dot\eta$ is chiefly determined by $\dot\gamma_T$ rather than $\dot\gamma$.

Another example is the so called *mixing factor* in the machines with the stochastic cooling.[2] It is basically the number of turns a particle needs to make in order to leave the original sample and is therefore *inversely proportional* to η. Stochastic cooling favors small values of the mixing factor, *i.e.* the highest possible η allowed by the amplifier passband. On the other hand, RF manipulations like the bunch rotation work best for small η. The optimal machine would have a variable η, such that, after the bunch rotation stage, η can be increased and the stochastic cooling proceeds with the optimal efficiency. Again, the way to achieve that is by changing the dispersion function in the dipoles.

Changing η may lead to new problems, most serious being a large increase of the maximal dispersion and, with it, the transverse emittance growth and the dynamic aperture limitations.

In this paper I describe the Local Dispersion Insert, a novel three-quadrupole cell which produces local dispersion- and beta waves while leaving all other beam parameters unchanged. In particular, since the desired γ_T jump can be achieved by locally *decreasing* the dispersion, the maximal dispersion of the accelerator remains unchanged. Therefore, the problems mentioned above are absent. This arrangement is extremely efficient. For example, in Fermilab's Main Injector, γ_T can be changed by one unit with as little as two triplets of low strength quadrupoles, while in the Antiproton Debuncher η can be changed by a factor of two by using the same number of quadrupoles. In the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, six triplets are sufficient to change γ_T by one unit.

In Section 2, the Local Dispersion Insert is described, while the Sections 3, 4 and 5 contain three applications of it: the γ_T adjustments in the Main Injector and the Antiproton Debuncher at Fermi National Accelerator Laboratory and in the Relativistic Heavy Ion Collider at Brookhaven National Laboratory.

2 Local Dispersion Insert

The Local Dispersion Insert and the corresponding lattice functions distortions are shown in Fig. 1. It consists of three lenses with the focusing strength ratio $-2:1:1$, π apart in the betatron phase. The net effect of the cell is the localized dispersion wave between the like sign lenses. There is also a localized beta wave inside the cell, however, the Local Dispersion Insert does not change the tunes of the machine.

The functioning of the Local Dispersion Insert can be explained as follows. A small perturbation of the gradient of a quadrupole causes a horizontal and a vertical tune shift, a horizontal and a vertical free beta wave and a free dispersion wave (in planar machines only horizontal). A local dispersion adjusting cell must have zero tune shifts and no free beta or dispersion wave escaping it.

The horizontal and vertical tune shifts caused by the small quadrupole perturbation $\Delta B'(s)$ are

$$\Delta\nu_x = \frac{1}{4\pi}\oint \beta_x(s)\frac{\Delta B'(s)}{B\rho}ds$$

and similar (of course, with the opposite sign) for $\Delta\nu_y$. For a thin quadrupole located at s_0,

$$\frac{\Delta B'(s)}{B\rho} = \Delta k\delta(s-s_0)$$

which gives

$$\Delta\nu_x = \frac{\Delta k \beta_x(s_0)}{4\pi}, \quad \text{and} \quad \Delta\nu_y = -\frac{\Delta k \beta_y(s_0)}{4\pi}.$$

The perturbations of the beta functions (beta waves) downstream from the (thin) quadrupole are

$$\frac{\Delta\beta_x(s)}{\beta_x(s)} = -\Delta k \beta_x(s_0) \sin 2\left(\mu_x(s) - \mu_x(s_0)\right)$$

and

$$\frac{\Delta\beta_y(s)}{\beta_y(s)} = \Delta k \beta_y(s_0) \sin 2\left(\mu_y(s) - \mu_y(s_0)\right).$$

The perturbation of the normalized dispersion function (dispersion wave) downstream from the quadrupole is

$$\frac{\Delta D_x(s)}{\sqrt{\beta_x(s)}} = -\Delta k D_x(s_0)\sqrt{\beta_x(s_0)} \sin\left(\mu_x(s) - \mu_x(s_0)\right).$$

Notice that the beta wave propagates with twice the frequency of the dispersion wave. No free dispersion wave will escape from a pair of quadrupoles of identical strength $(2n+1)\pi$ apart in phase, however, such arrangement will produce beta waves. Similarly, no free beta wave will escape from a pair of quadrupoles of equal and opposite strengths $n\pi$ apart. Finally, no dispersion wave is created by a quadrupole in a zero dispersion region. By combining these three statements, we arrive at the Local Dispersion Insert: two quadrupoles of equal strength π apart in phase accompanied by another one with the double and opposite strength, π (or $n\pi$ if necessary) apart and in a zero dispersion region. It is obvious from the above equations that the tune shifts are zero, the beta wave is localized in the interior of the cell and the dispersion wave is non zero only between the like sign quadrupoles.

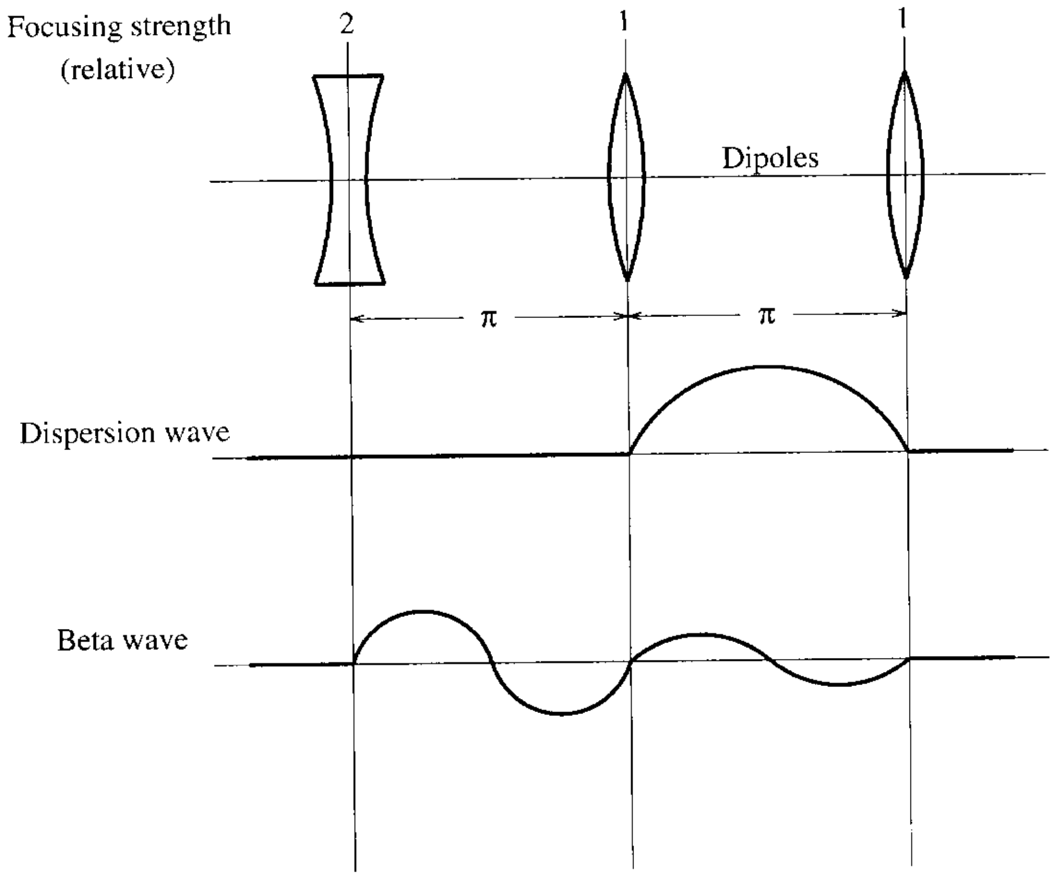

FIGURE 1 The Local Dispersion Insert and the corresponding dispersion and beta waves.

3 Local Dispersion Insert in the Main Injector

It has been widely acknowledged for several years now[3] that the transition crossing in the Main Injector is the main problem for this accelerator. The Local Dispersion Insert is the solution of the problem.

The phase advance per cell in the Main Injector is $\pi/2$. The lattice functions of one half of the machine are shown in Fig. 2. See Ref. 4 for various γ_T jump schemes proposed so far.

The natural position for the Local Dispersion Insert is on each side of a long straight section shown in Fig. 2. The horizontal beta function has maxima of the same size at the quadrupoles marked 1, 2, and 3, which are also π apart. In addition, the dispersion function is zero at the position of the quadrupole 1. Consequently, adding Δk to the quadrupoles 2 and 3 and $-2\Delta k$ to the quadrupole 1 will create a dispersion bump between the quadrupoles 2 and 3, and a beta wave between the quadrupoles 1 and 3. There will be no changes in the remaining part of the ring.

Figures 3 and 4 beautifully illustrate the effect of the Local Dispersion Insert: the dispersion is increased in the Inserts, while it is unchanged in the rest of the machine. As predicted, $\Delta\nu_x = \Delta\nu_y = 0$. Since $\Delta\gamma_T > 0$ is achieved by locally *decreasing* the dispersion, the maximal dispersion does not change, however, if we wish $\Delta\gamma_T < 0$, the local dispersion wave will add to the maximal dispersion, see Table I.

Several important lattice parameters are shown in Table I for $\Delta\gamma_T$ of 0.5, 1 and -0.5.

TABLE I

$\Delta\gamma_T$	Max β_x(m)	Max β_y(m)	Max Dispersion	$\Delta\nu_x$	$\Delta\nu_y$
0	59.1	62.7	1.95	–	–
0.5	77.3	64.5	1.98	$< 10^{-3}$	$< 10^{-3}$
1.0	172.9	72.5	2	$< 10^{-2}$	$< 10^{-2}$

4 Local Dispersion Insert in the Antiproton Debuncher

Although in the Antiproton Debuncher at Fermi National Accelerator Laboratory there is no transition crossing, it would be desirable to have a "γ_T *knob*" as explained earlier. During the bunch rotation phase small η is preferred, while in the stochastic cooling stage it is desirable to have the mixing factor[2]

$$M = \frac{p_0 f_0 \psi(p) \ln \frac{f_{max}}{f_{min}}}{2\eta W N}, \tag{2}$$

as small as possible. Here, p_0 is the particle momentum, f its revolution frequency, $W = f_{max} - f_{min}$ the amplifier bandwidth, and N the number of particles in the beam. Practically η is limited (by the width of Schottky bands) to an increase of about factor of two.

The phase advance per cell in the Debuncher is $\pi/3$. The lattice functions of one sextant of the machine are shown in Fig. 5. The natural position for the Local Dispersion Insert is again on each side of a zero dispersion straight section. The efficiency of the Local Dispersion Insert is clear from Fig. 6 – a modest change of the quadrupole strength changes η by the required factor of two.

5 Local Dispersion Insert in the Relativistic Heavy Ion Collider

The lattice functions in one superperiod (one third) of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory are shown in Fig. 7. The phase advance is close to $\pi/2$ per cell. It is straightforward to install two Local Dispersion Inserts in the first sextant. Figs. 8 and 9 show their effect: the dispersion is decreased or increased in the Inserts, while it is unchanged in the rest of the machine. The relevant beam parameters are shown in Table II. The noticeable but unimportant perturbation of the beta function is due to the fact that the phase advance per cell is not exactly $\pi/2$ and the cancellation of beta waves outside the Inserts is not complete.

TABLE II

$\Delta\gamma_T$	Max β_x(m)	Max β_y(m)	Max Dispersion	$\Delta\nu_x$	$\Delta\nu_y$
0	144	143.7	1.84	–	–
0.5	166.5	145.5	2.08	$< 10^{-3}$	$< 10^{-3}$
-0.5	219.3	149.2	2.38	$< 10^{-2}$	$< 10^{-2}$

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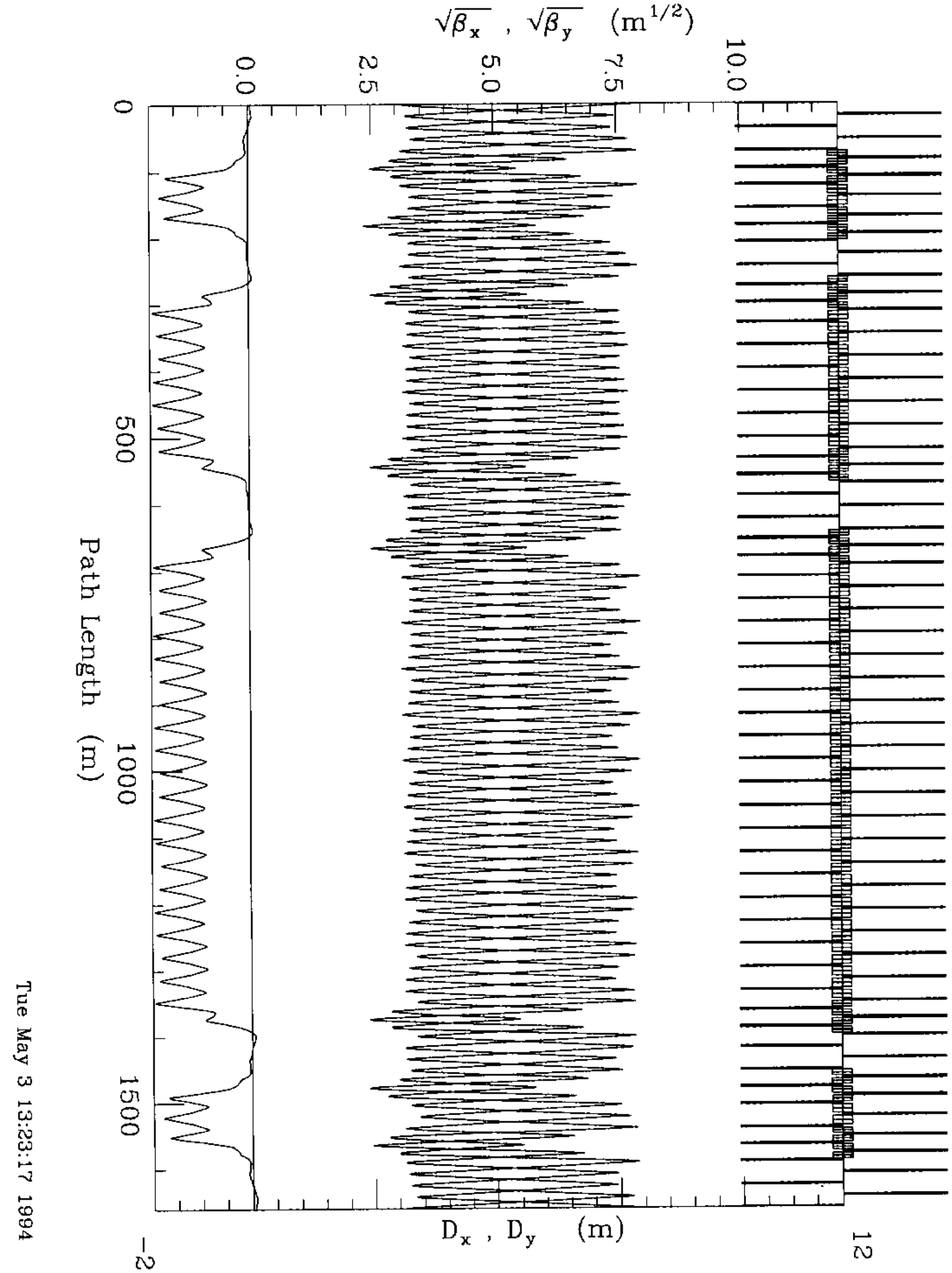

FIGURE 2 The lattice functions in one superperiod of the Main Injector at Fermi National Accelerator Laboratory. $\gamma_T = 21.59$.

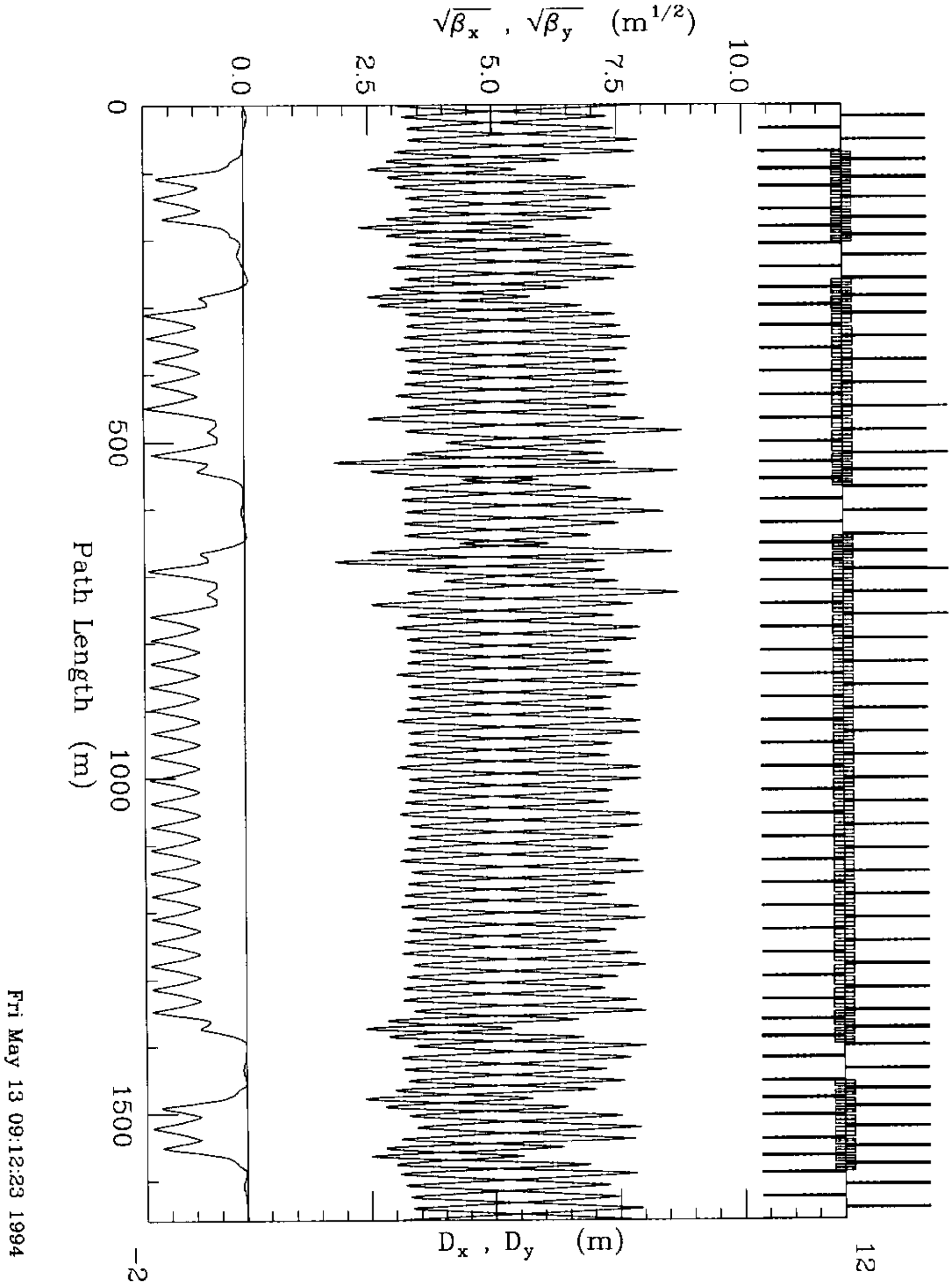

FIGURE 3 The lattice functions in the Main Injector with the Local Dispersion Insert. $\gamma_T = 22.07$

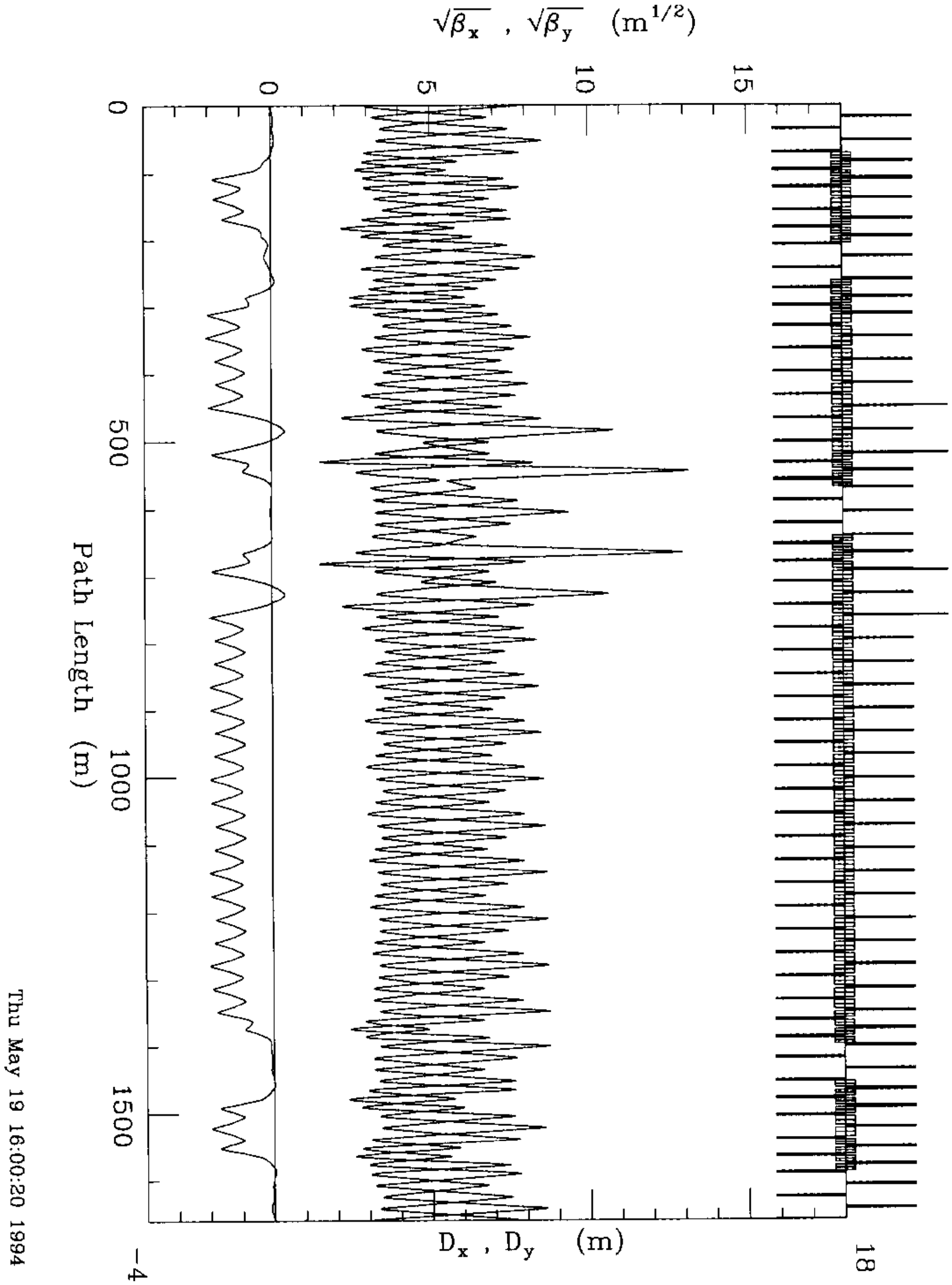

FIGURE 4 The lattice functions in the Main Injector with the Local Dispersion Insert. $\gamma_T = 22.59$

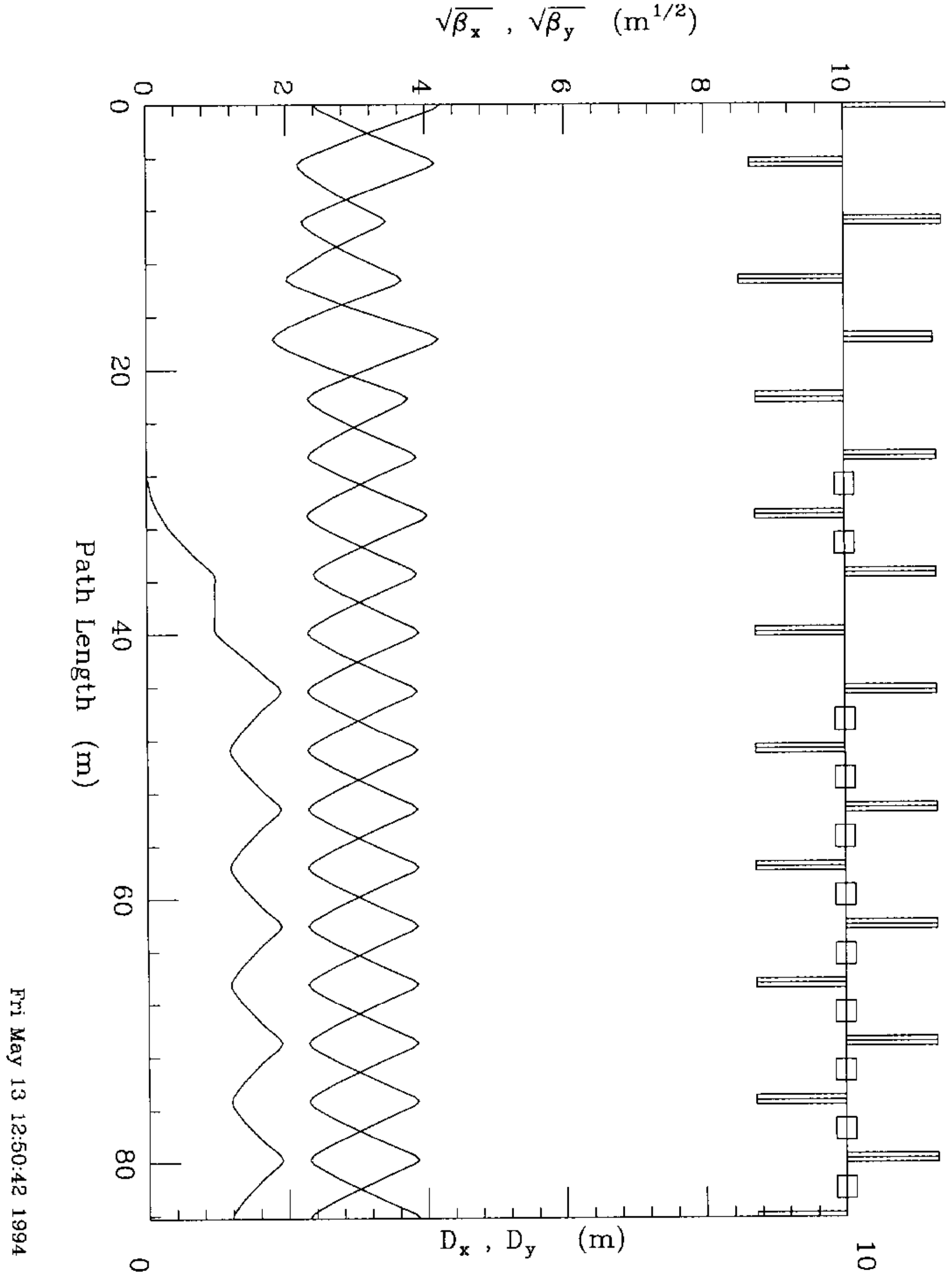

FIGURE 5 The lattice functions in one superperiod of the Antiproton Debuncher at Fermi National Accelerator Laboratory.

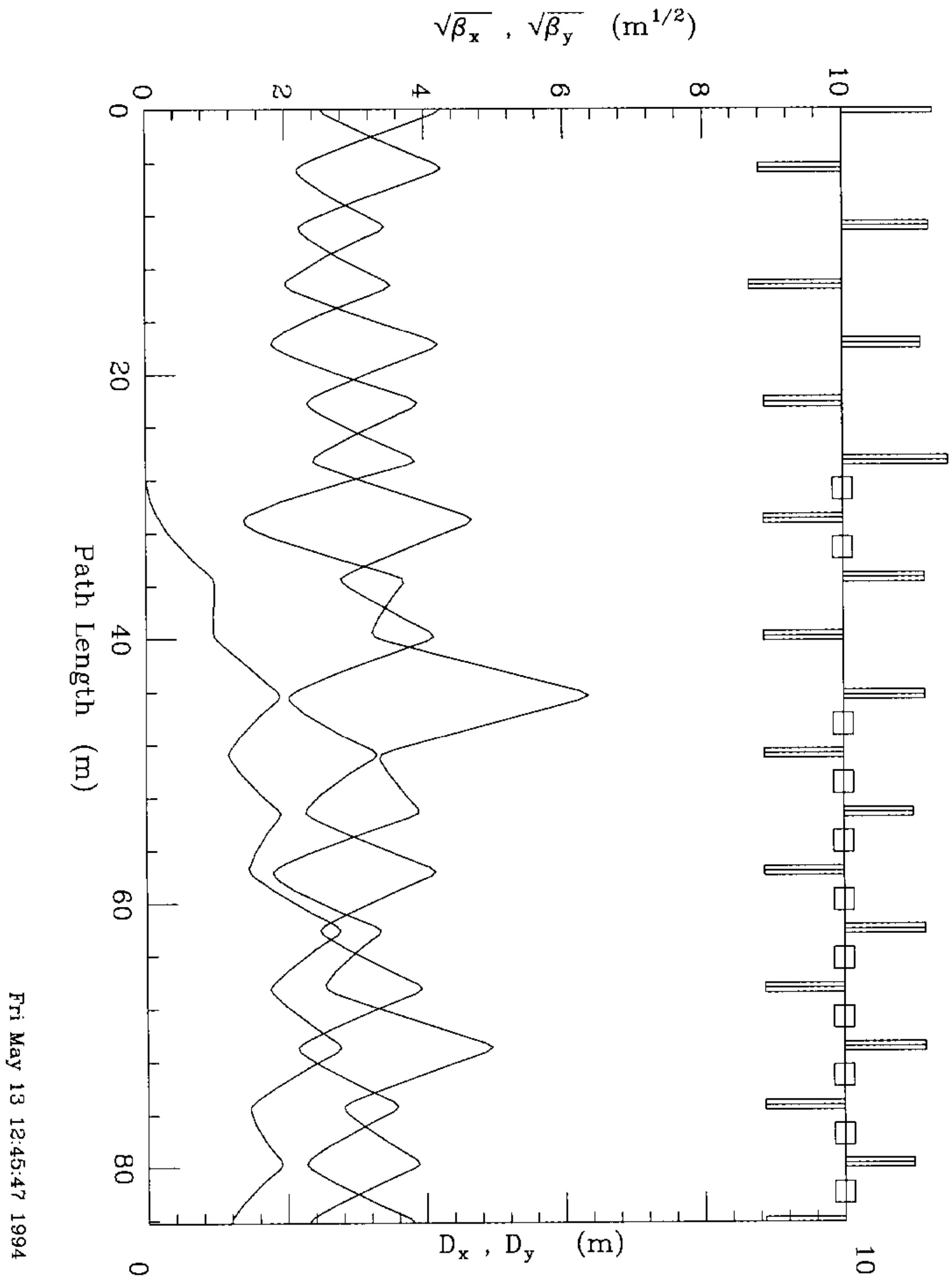

FIGURE 6 The effect of the Local Dispersion Insert in the Debuncher Lattice. η is doubled compared to the lattice of Fig. 5.

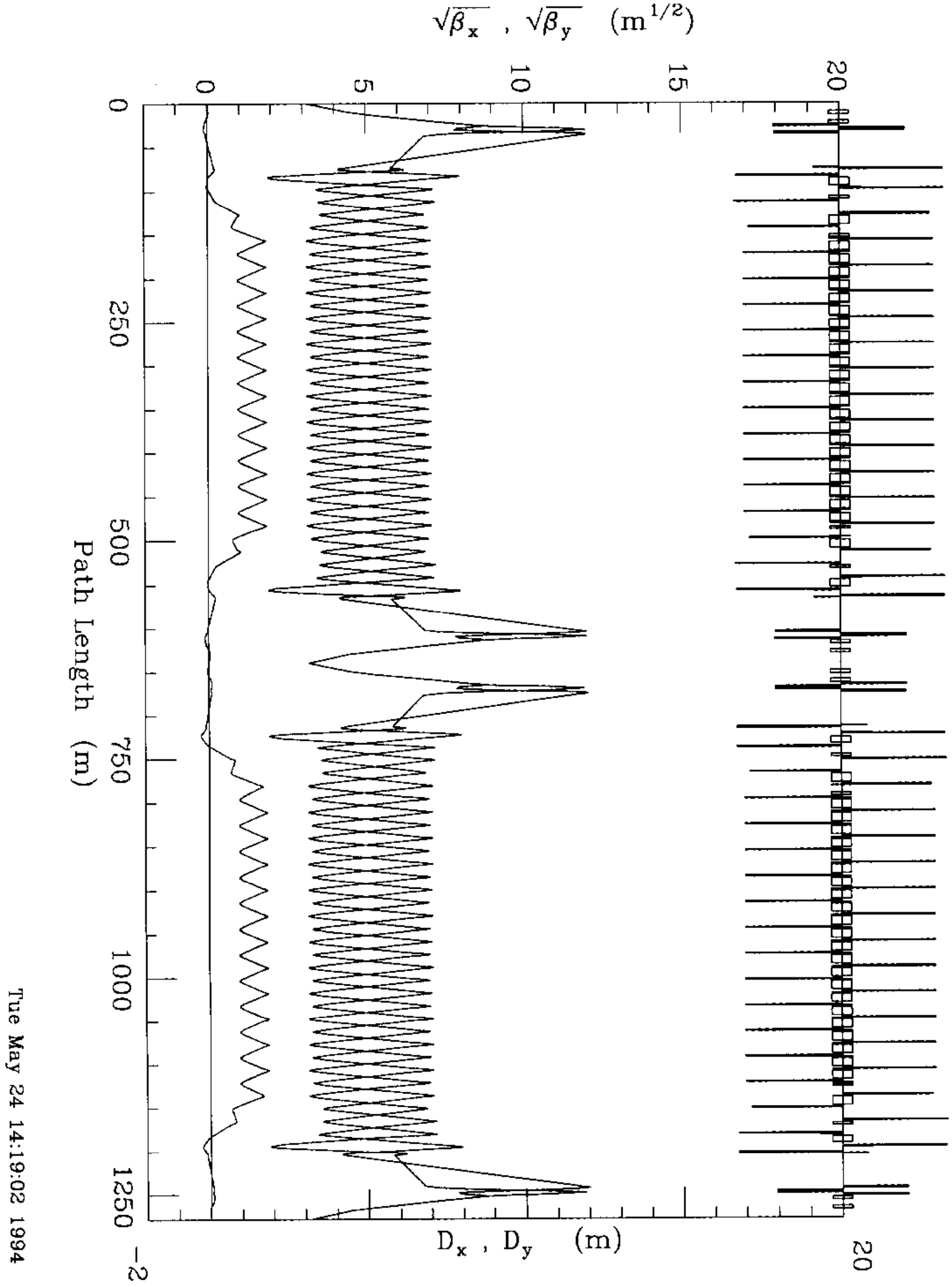

FIGURE 7 The lattice functions in one superperiod of the Relativistic Heavy Ion Collider at Brookhaven National Laboratory. γ_T = 22.9 and the maximal value of the dispersion is 1.84 meters.

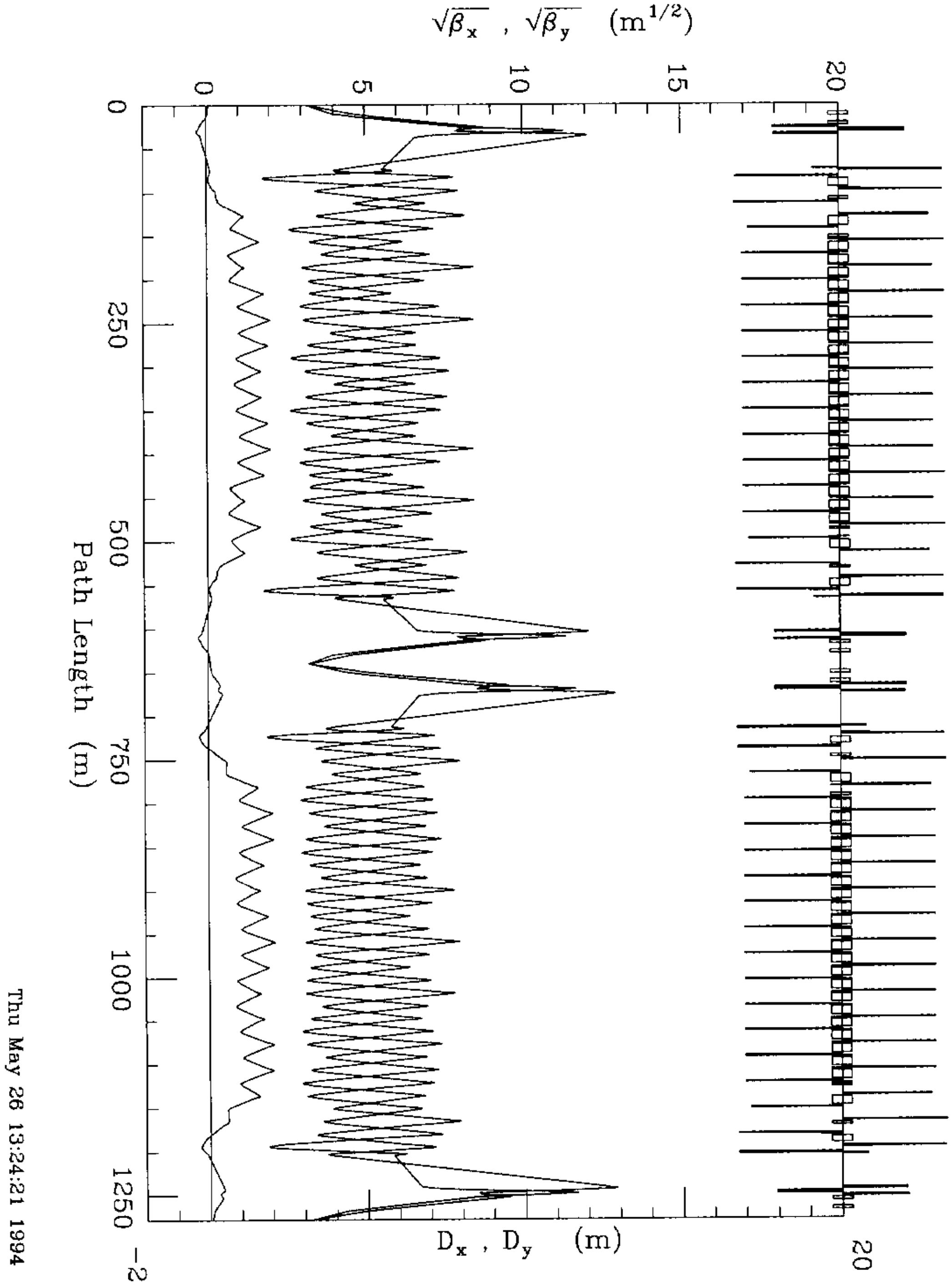

FIGURE 8 The effect of the Local Dispersion Insert in the RHIC lattice. γ_T is increased by half a unit to 23.4. The maximal value of the dispersion is 2.08 meters. Due to imperfect cancellations of the beta waves the maximal beta function increases from 144 to 166 meters.

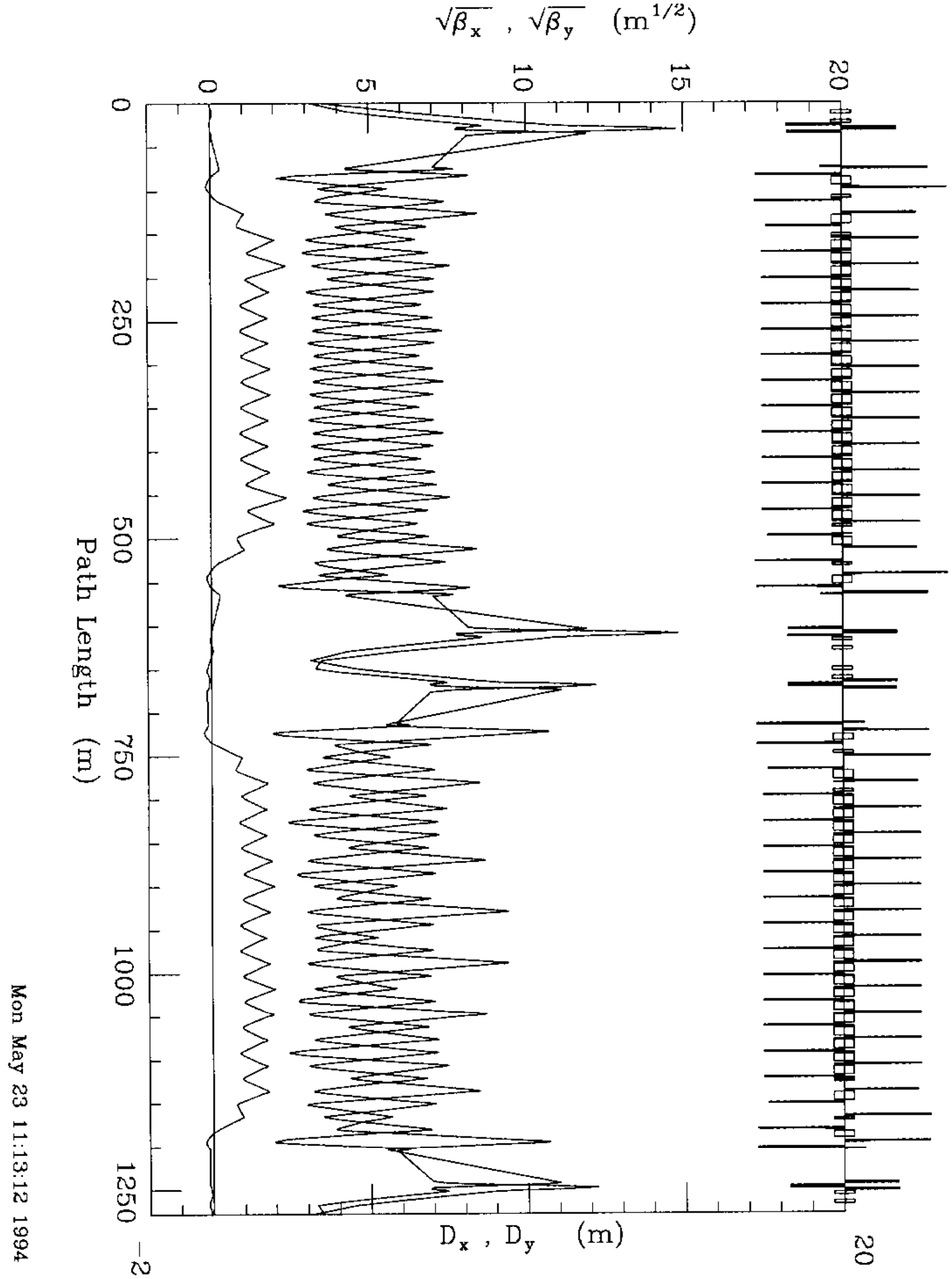

FIGURE 9 The effect of the Local Dispersion Insert in the RHIC lattice. γ_T is decreased by half a unit to 22.4. Due to imperfect cancellations of the beta waves the maximal beta function increases from 143 to 219 meters.